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Abstract

This paper studies price and service co-opetition of remanufacturing considering uncertain demand and uncertain condition of the acquired items. Two firms considered here are competing on price and service to sell their substitutable products through a common retailer and both the firms provide service directly to the end customers. Mathematical models are developed for four different remanufacturing configurations and derive the equilibrium decision and profit under each of the configurations. Further, identify the condition under which the manufacturers can select direct system and integrated system compared to global system. One of the main findings is that, direct system provides better result in terms of channel profit compared to other channel configurations under absence of price and service competition.

Keywords: Price, service, coordination, competition, remanufacturing, co-option

1. Introduction

Today's competitive business environment and growing concern of people about the adverse effect of end-of-life products on its environment has drawn the attention of academia and the industry practitioners on remanufacturing. Consumers concern and social responsibility of corporate towards environment, awareness about the limited natural resources, and government legislations are some of the important drivers for remanufacturing. The economic potential is also another important key driver that attracts many manufacturers to join in remanufacturing. As a result, in recent times, more number of remanufactured products is available in the market. The remanufacturing firms while collecting the used products from the market face a significant challenge due to the variation in the conditions of the used products and uncertain returns of it. The remanufacturer can manage variation while acquiring the product externally. Acquisition quantity is a key decision for the remanufacturer and has direct impact on the profit of the firm. In U.S., more than 73,000 firms participate in remanufacturing and take some role in closed-loop supply chain (CLSC) while about $53 billion remanufactured products are sold annually in the market (US EPA, 1996). In India, the rate of return of used product is 5% costing US $12 -15billion (Thomas, 2012). Government of India has made a target to recover 1.5 million tonnes of steel scrap, 180,000 tonnes of aluminum scrap and 75,000 tonnes of plastics and rubber like materials through recycling by 2020 (Business today, 2013).

In the current globalized business environment, pricing has become an important competitive tool and the competing firms often play a price war to attract customers. Apart from price, nowadays, customers are also equally interested on service and quality of the products. Therefore, besides pricing, service also influences the buying decision of customers. For example, in auto industry, financial services such as auto loan, insurance, and maintenance service play an important role in selecting a brand for the customers (Xiao and Yang, 2008). Even the customers show their concern about the post sales service while purchasing mobile phone, printer, cartridge, computers etc. Similar is the case of remanufactured products. When a customer goes for purchasing such remanufactured products, the customer looks for the service for such type of products. The large personal computer manufacturer HP inc. has adopted a remanufacturing program called HP renew program for recycling and selling the remanufactured or refurbished products (Wu, 2012; Wei et al., 2013). Its remanufacturing program certifies that the remanufactured products perform as like as new and well substitute the new products at lower prices and also provides service for the remanufactured products. Consumers can choose between the remanufactured and brand new products based on product prices and service offerings. Many real life examples are available such as cartridge, automobile, camera, electronics item etc. One major research issue is how should the remanufacturer interact with the end customers for providing the service and determine price in the competitive environment?

In recent times, a new business trend has been observed among the competing firms where they collaborate on certain issues to improve their performance in terms of cost, delivery or customer satisfaction. In the literature, this phenomenon is termed as co-opetition. Brandenburger and Nalebuff (1996) have mentioned that co-opetition is a portmanteau of cooperation and competition. Co-opetition takes place among the players when they are in the same market working together for the exchange of knowledge and research of the new products and at the same time competing for their market share. Since remanufacturing is a comparatively new area, competing firms can exchange their knowledge or research output with their competitor for the benefit of both the parties.

Most of the earlier studies have examined price and service competition among the players considering deterministic demand ignoring uncertain market demand and condition of the used items. Only Galbreth and Blackburn (2006) have analysed the optimal acquisition and sorting policies in the presence of used products condition variability for a remanufacturer under deterministic and uncertain demand situation. They have provided solution by making tradeoff between acquisitions cum scrapping cost and remanufacturing costs when the used product condition varies widely and multiple re-manufacturable conditions are possible. They have not considered competition among the players under the uncertain demand and condition of the used items return. A research question needs answer is how the profit of CLSC will be affected when there is competition among the players under stochastic situations. To bridge this gap in the literature, in this paper, price and service competition are studied considering uncertain demands and condition of the acquired items simultaneously. A new dimension called co-opetition among the competing firm is also incorporated in the present study and thereby, it is different from all the related studies made earlier and has provided new contributions in the existing literature. While developing the models, we have considered that two different remanufacturers are producing substitutable products and compete on price and service to sell the products individually. The primary goal of this research is to understand the impact on different remanufacturing configurations when the remanufacturers encounter uncertain condition of acquired items and market demands under price and service competition. Two alternatives channels are considered here for selling the remanufactured products and they are called as indirect and direct system. In the indirect system, they sell their product through a common retailer. The indirect system is studied for three different configurations where (i) All the three members individually maximize their profit competing on price and service (called as Decentralized system), (ii) Both the remanufacturers coordinate separately with the retailer and both the integrated channels compete with each other on choosing the price and service level to maximize the channel profit (termed as Channel wise integrated system)and finally (iii) All the members collude to form a global system (Global system). On the other hand, in the direct system, both the remanufacturers sell their products directly to the market. Mathematical models are developed for all the above-described configurations and optimal selling price, level of service and quantity to be acquired are derived to maximize the total profit of each system. Here, we have also identified the condition under which the direct system is better than the indirect system. Finally, quantity discount with return contract has been developed to coordinate the decentralized system.

The paper is organized as follows: A brief review of literature is included in Section 2. Section 3 is devoted to the key assumptions and notation of the modeling framework. The mathematical models are developed in section 4, numerical example is described in Section 5.In Section 6, coordination mechanism is studied between the remanufacturer and the retailer and sensitivity analysis, and managerial insights drawn from the study are presented in Section 7. Finally, conclusions and future scope of the study are presented in Section 8.

2. Literature review

Keeping in view of our research work, the review of literature is divided into two sections (i) Literature related to uncertain demands and condition of returned items (ii) Secondly, literature related to competition. Both research streams are discussed briefly in the following sub-sections.

2.1 Literature related to uncertain demands and condition of returned items

The issue of the uncertainty in remanufacturing has been attracting more attention in recent literature. Uncertainty in demands and returns are appeared because of the inexact predicting of demands and returns. Inderfurth (2005) addressed uncertainty in demands and returns are considerable obstacle to following an appropriate CLSC network under a reverse logistics system. They developed a model for making cost-efficient decisions on the product recovery behavior considering uncertainty inputs such as return, demand and yield etc. Mukhopadhyay and Ma (2009) addressed the issue of a hybrid system where used and new parts are taken as inputs in the remanufacturing process to satisfy the uncertain market demands. They developed a mathematical model taking into the account of a single product for a single period to analyze the firm's production and procurement decisions under quality and demands uncertainty in the closed-loop supply chains. Pishvaee and Torabi (2010) addressed that the uncertainty in demands, return, costs and capacities are considerable obstacles for determining the collection of used items, number of distributions and recycling centers under multi periods setting. They developed a mathematical model using mixed integer linear programming to minimize the total costs and minimize the total delivery tardiness. Shi et al.(2011a)studies a hybrid system in the sense of manufacturing brand-new products and remanufacturing returns as like as new one with pricing and production decisions, that satisfy the market demand. They developed a mathematical model considering uncertainty in demands and returns for single product to maximize the overall expected profit of the system. Shi et al. (2011b) considering multi-products CLSC under stochastic demand and returns relax this assumption. Hasani et al. (2012) studied a robust closed-loop supply chain network design model for perishable goods in agile manufacturing system. Authors developed a mathematical model considering uncertain demand and uncertain purchasing cost for food and high tech electronics manufacturing industries. This model is considered various assumptions such as multiple periods, multiple products, and multiple supply chain echelons. Saman and Zhang (2013) proposed a model to select the best suppliers, remanufacturing subcontractors, and refurbishing sites based on quantitative and qualitative criteria and in an uncertain demand.

In the literature, variability in the quality of return products has been well documented (Bloemhof-ruwaard et al., 1996). Guide et al.,(2003) studied the impact of condition variability on used product acquisition decisions. They have developed a model taking into the assumptions of price sensitivity demands and return rates. However, authors controlled the quantity, quality and timing of returns by the price offered to buy back used items. Galbreth and Blackburn (2006) modify this work for analyzing the optimal acquisition and sorting policies in the presence of used products condition variability for a remanufacturer under deterministic and uncertain demand situation. But their model assumed that the condition distribution of an acquired is known with certainty. Galbreth and Blackburn (2010) considering the condition of uncertainty for acquired items relax this assumption. They have provided solution by making tradeoff between acquisitions cum scrapping cost and remanufacturing costs when the used product condition varies widely and multiple re-manufacturable conditions are possible. Fleischmann et al. (1997) evaluated several reverse logistics configurations taking into the account of uncertainty in the quality of return products. In this paper, authors proposed a model with mixed integer linear programming recovery network design considering generic characteristics of a number of case studies. Danizel et al. (2010) assumed that the grading of return items is random in each period, and thus devolved a multistage stochastic model based on the number of returns obtained from each quality grade. They presented the advantages of stochastic programming formulation, as it detects infeasibilities that are not possible in the deterministic case. Souza et al. (2008) addressed the problem of allocating returns of different quality grades to maximize profit considering a required service level measured in terms of flow times. In this paper, a significant stream of research is developed considering hybrid inventory management system where remanufactured and new products are perfect substitute. In this paper, we have also considered same assumptions under co-opetition strategies. Wang and Li (2012) studied dynamic quality-based pricing model with single and multiple markdown policies and assumed that customers are sensitive to product price and the remaining shelf life of the product. Authors addressed that once a product has produced, the retailer has to reduce the price over time to make sure that the partially deteriorated products are still bought by the customer. Aras et al. (2004) were the first to make analytical study considering explicitly quality categorization of used items in the context of hybrid manufacturing and remanufacturing firm. In this paper, they investigated the issue of stochastic nature of the returns and also find condition under which quality based categorization is most effective.

Kim et al. (2013) studied how the use of RFID impacts on the return of the containers that are returned to the supplier is stochastic. They developed a mathematical model to analyze under what condition the use of RFID is economical. Authors found that the use of RFID-tagged containers can be justified with higher purchasing cost if they improve the predictability of the containers flow. Several models have discussed about the acquisition decision when the condition variability of used product is fully captured by two categories i.e., re-manufacturable or not (Zikopoulos and Tagaras, 2007; 2008). They studied a sorting procedure with limited accuracy that classifies items as good (i.e. re-manufacturable) or bad (i.e. non-remanufacturable). They have also determined the conditions under which it is optimal. In this paper, we model price and service decisions considering in demands and conditions uncertainty under price and service competition. Our model provides results that are relevant for remanufacturing practices under co-opetation strategy.

2.2 *Literature related to competition and coordination*

From the perspective of economic theory, a large number of research papers are available on market competition. Most of these papers deal with either quantity competition or price competition and their primary focus are on applying game theory to derive equilibrium under varied assumptions. Gronevelt and Majumdar (2001) developed a model of remanufacturing in the face of competition between original equipment manufacturer and local remanufacturer under different reverse logistics configurations for the returned items. They formulated a two-period model to examine the effect of competition in remanufacturing system. In the first model, manufacturer manufactures and sell the new items where as in second period, a fraction of the items are returned for the remanufacturing. Boyachi and Gallego (2004) studied a market with two competing supply chains where each consists of one wholesaler and one retailer considering three competition scenarios. Authors assumed that each supply chain charges similar price and to compete on the basis of customer service. They developed a game theoretic model to study customer service competition. Bernstein and Fedegruen (2005) examined the equilibrium behavior of decentralized supply chain with competing retailers and a single supplier under demand uncertainty in two-echelon distribution systems. They design a contractual arrangement between the parties for performing decentralized and centralized chains. Savaskan and Van Wassenhove (2006) addressed the interaction between manufacturers reverse channel choice for collecting used products and pricing decision in the forward channel under retail competition. In this paper, Authors studied the direct and indirect collection of closed-loop supply chain. They examined the effect of collection of used products on the new products market. They found that in direct reverse channel, the total channel profit is driven by the impact of returns on the collection effort, whereas in the indirect reverse channel, the total chain profit is driven by the competitive interaction between the retailers. But, here we have considered price and service competition between the remanufacturers under co-optetion strategy. Chen and Chang (2012) studied the management of two differentiated version of the similar products. Authors developed an analytical model and observed that the pricing strategy typically depends upon the types of market. Mitra and Webster (2008) studied the price competition between new and remanufactured product in the market under the effect of government subsidies. In this paper, they analyzed the effect of government subsidies on the remanufacturing system. This paper found that manufacture's profit in the remanufacturing activity decreases as the government subsidies increases. Ferguson and Toktay (2006) developed a model to support a manufacturer's recovery strategy in the face of competitive threat on the remanufactured market. This study examined the competition between new and remanufactured product produced by a manufacturer and identify the favorable condition under which the firm would not take an interest for remanufacturing. Gu and Gao (2012) and Jena and Sarmah (2014) addressed the wholesale price, the retailer price and collection price of the used product for two competitive CLSC. Further, Wu (2012) studied price, service competition between the manufacturer, and remanufacture with a common retailer. The author has investigated the profits of the chain members by considering different interactions between price and service. Benitez and Muriel (2014) studied the wholesale price and buy-back contract between a manufacturer and retailer considering retailer price exogenously under stochastic demand. Whereas, Xiao and Gilbert (2007) have shown that the linear quantity discount scheme can coordinate the supply chain with two competing retailers. Qiang et al. (2013) addressed a network model with decentralized decision makers consisting of raw material suppliers, retailer outlets, and the manufacturer that collects the used products from the market. They proposed an algorithm that can allow for the discussion of the effects of competition, uncertain yield and demand uncertainty on equilibrium quantity transactions.

2.3. Synthesis of both research streams

The literature review reveals that how different remanufacturing configurations are affected when the members encounter uncertain condition of acquired items and market demands under price and service competition. Galbreth and Blackburn (2010) is the only paper that discussed about the impact of condition of used items on total cost. However, the paper has limitation as it studies a monopoly scenario. Another paper by Savaskan and Van Wassenhove (2006) have studied only the interaction between a manufacturer's reverse channel choice to collect the used product and a strategic product pricing decision in the forward channel in retail competition. Wu (2012) discussed price and service competition between the manufacturer and remanufacturer with a common retailer. But it has limitation as it considered deterministic scenario. But all these literatures have not discussed about price and service co-opetition of remanufacturing under uncertainty scenario.

The purpose of this paper is to link the two research streams reviewed above and to propose a method for selling the products in a co-opetition under uncertain demand and condition of return. This paper has discussed the impact on different remanufacturing configurations when the remanufacturers encounter uncertain condition of acquired items and market demands under price and service competition.

3. Model overview

3.1. Key assumptions and notation

The following notation in Table 1 are used for the development of the mathematical models (i,j=1, 2, $i \neq j$). Here, we used the symbol g, I, ds and D for global, channel wise integrated, decentralized and direct system respectively.

Table1: Notation

Symbol	**Description**
D_i	Demand of products per unit time at remanufacturer *i*
Q_i	Quantity of products ordered by retailer from remanufacturer *i*
λ	Used products condition
w_i	Unit wholesale price of remanufactured product at remanufacturer *i*
p_i	Unit selling price of remanufactured products of remanufacturer *i*
$z_i =$	$Q_i - (\alpha_i - \beta_p p_i + \gamma_p (p_j - p_i) + \beta_s s_i - \gamma_s (s_j - s_i))$ [38,12]
s_i	Service level of remanufacturer *i*
η_i	Service cost coefficient of remanufacturer *i*
f_i	Unit acquisition and inspection cost of used items is made by remanufacturer *i*.
S	Shortage cost per unit
a_i	Fixed cost of remanufacturing of remanufacturer *i*
c_i	Variable cost of unit remanufacturing of remanufacturer *i*
$F(\lambda)$	Condition CDF
$f(\lambda)$	Condition PDF
$g()$	Probability density function of ε
$G()$	Cumulative distribution function of ε
n	Index for order item condition within an acquired lot
u	Buyback price per unit
Π_r	Profit of the retailer
Π_{Rmi}	Profit of the remanufacturer *i*
Π_I	Profit of the integrated system
Π_g	Profit of the global system
Π_T	Profit of the total system

Here, the model development follows the similar line of Galbreth and Blackburn (2006) where the remanufacturer fulfills the market demand after getting the order from the retailer. For example, toner cartridge remanufacturer produces cartridge based on the order received from an office supply store. This model is applicable across a variety of large remanufacturing industry such as cell phones, power tools etc. The remanufacturers can produce the product directly from the used materials or from the remanufactured part and the customers (Petruzi and Dada, 1992) treat the remanufactured products as like new products. Each remanufacturer produces only one-type products, and both the firms want to enhance the demand by providing after sale-service such as maintenance and warranty repair agreement. Like Ferguson et al. (2009), we have also considered the condition λ of each used products as a real number $\lambda \in [0, 1]$, the value of which is a random variable with density function $f(\lambda)$ and distribution function $F(\lambda)$. In this model, best and worst possible condition of the acquired items are represented by $\lambda=0$ and $\lambda=1$ respectively. Remanufacturing cost is an increasing function of λ. The remanufacturer *i* collects R_i units of used items from the market against the order of retailer Q_i, where $R_i \geq Q_i$ because of uncertain condition of acquired items. For example, a remanufacturer might face a demand of 500 cell phones, to avoid shortage; at least 500 must be acquired because of uncertain condition of used items (Galbreth and Blackburn (2006)). When R_i used items are acquired, the condition of the best Q_i items are uniformly distributed over $\left[0, \frac{Q_i}{R_i}\right]$. All the acquired used items are inspected and ranked by order condition, allowing the best Q_i condition items to be remanufactured. Unit acquisition and inspection cost of used items are f_i .The acquiring cost in all the methods of collection includes the cost of dismantling, inspection, quality assurance and other management costs. For the proposed model, we have assumed that remanufacturing cost is a linear function of condition and represented by $a+c\lambda$, where a is known, fixed unit cost (for disassembly, cleaning, etc.) and c is a variable component that depends on condition. Let $X_{(1)}, X_{(2)}, \ldots\ldots X_{(Q)}$, denote the order of condition of the Q_i items. The remanufacturing cost of all the items for the remanufacturer *i* is given by $\sum_{n=1}^{Q}(a_i + cX_{(n)})$. For uniform distribution, the n_{th}-order used item has the following density function λ

$Q_i\binom{Q_i-1}{n-1}\lambda^n(1-\lambda)^{Q_i-n}$; $\lambda \in [0,1]$(See Galbreth and Blackburn (2006)).

The demand of the product is stochastic, volatile, and considered price and service-sensitive. If the realized market demand of retailer is higher than his initial order quantity, the retailer will face lost sales with additional penalty cost. Especially in electronic products, the remanufacturers have taken less interest to keep inventory because of high rate of obsolesce and rapid fall of price over time. Therefore, the model has been developed considering two periods only. First period is collecting the used products from the market and second period is for selling the remanufactured products into the market. Considering demand D_i and order quantity Q_i, the expected number of sales, left over products and the shortages at the end of the selling season are given by $E\left[\min(z_i,\varepsilon)\right]^+$, $E[Q_i - D_i]^+$ and $E[D_i - Q_i]^+$ respectively.

Following are the assumptions in the development of the model:

Assumption 1: The demand functions of the substitutable products produced by two remanufacturers are continuous, stochastic, price and service sensitive and assumed to be of the following form, i.e., $D(p_i,\varepsilon)=\alpha_i-\beta_p p_i+\gamma_p(p_j-p_i)+\beta_s s_i-\gamma_s(s_j-s_i)+\varepsilon,\ (i,j=1,2,i\neq j),$

Where $\alpha_i>0,\beta_p>0,\gamma_p>0,\beta_s>0,$ and $\gamma_s>0.$ The stochastic demand quantity (ε) can be defined in the range [A, B]. The value of A is usually zero, B is a value far larger than zero, and probability distribution, e.g. uniform distribution, can provide adequate approximation.

This demand function is a variation of more general class linear demand functions used in many aforementioned studies (McGuire and Staelin, 1983; Choi, 1991). The two parameters β and γ are independent in nature. It is assumed, α_i, $\beta_p,\gamma_s>0>0$ so that demand for each product decreases with the increase in price and demand for each product increases with service level. For remanufacturer *i*, α_i is measure of the size of the remanufacturer *i*'s market when both prices are at zero and no service is offered. β_p and β_s represent the price and service elasticity. Whereas γ_p and γ_s measure the price and service competition between the two remanufacturers. Here, we assume that both the remanufacturer *i* and *j* are the Stackelberg leader and sells remanufactured products to the retailer at the same whole sale price w_i and w_j due to undifferentiated product between new and remanufactured products. The retailer sells the product in the market at price p_i and p_j respectively.

Assumption 2: The demand structure is symmetric between the two products. Demand for a product is decreasing in its own retail price and increasing with the competitor's retail price. On the other hand, demand for a product is increasing in its own service and decreasing with the competitor's service.

Assumption 3: In all the systems, remanufacturers must acquire returned products equal or more than the quantity ordered by the retailer to avoid shortage. As a result the number of excess items acquires will increase which helps the remanufacturers to opt for more selective and remanufacture of those items that are in best conditions. The values of left over used items are assumed to be zero for the remanufacturer due to worst condition of left over items. Therefore, for simplicity, it is considered that remanufacturer must acquire equal quantity as demanded under different remanufacturing cost.

Assumption 4: Due to uncertainty of demand, the retailer may face the overstocking of remanufactured products. To avoid overstocking, the retailer sells the entire unsold products to other customer with a low salvage price.

4. Model development

We have developed mathematical models considering (a) Indirect system and (b) Direct system. In the indirect system, the remanufacturer sells the product through the retailer where as in direct system; the remanufacturer sells the products directly to the market. As mentioned earlier, under indirect system, three different configurations of remanufacturing are considered (i) Global system (ii) Channel wise integrated system and (iii) Decentralized system. In the following subsections, first, we discuss different configurations of remanufacturing for the study under the indirect system.

4.1. *Global system*

In this case, both the remanufacturers and the retailer act together as single entity to maximize the total profit of the system. This global system is shown in Fig.1a. Both the remanufacturers' collect the used product from the market at a certain price f_i and f_j respectively for different quality of products and then sorted based on the condition of the used products. The return price depends on the condition of the used products. The whole system is providing service s_i and s_j to the end customer directly. The global selling price and service level are determined simultaneously among the players. In the global system, each individual member tries to maximize the whole system profit ignoring the impact of his decision on his or her own profit.

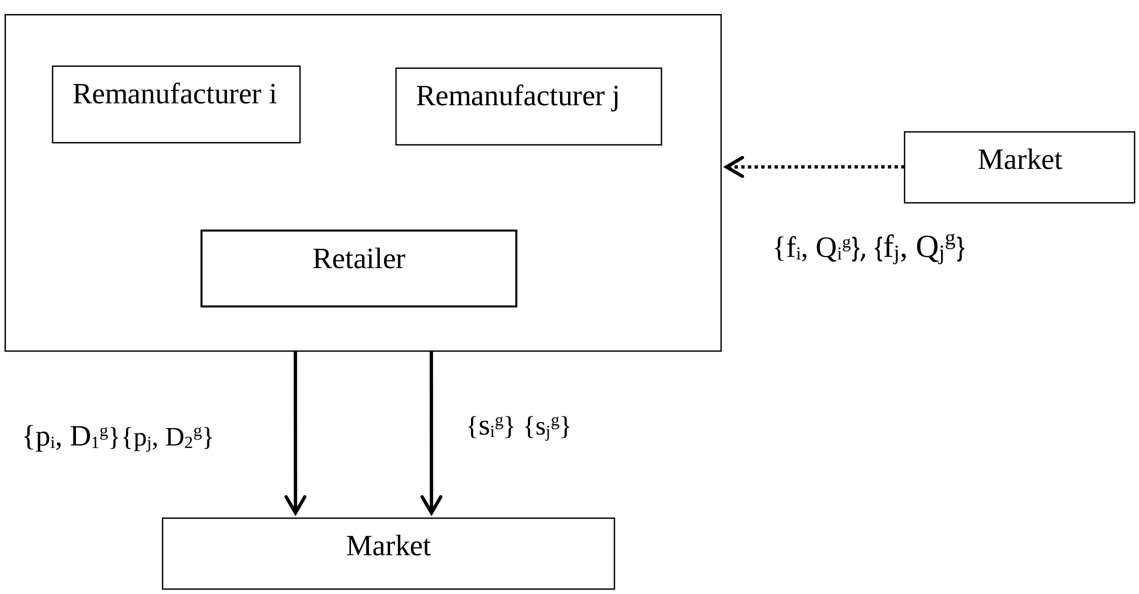

Fig.1a.Global system

Theorem 1

When the condition of used items is uncertain and remanufacturing cost is a linear function of the condition of the used items, the total remanufacturing cost is given as $\left(a_i + \frac{c_i}{2}\right)Q_i$

Proof: See Appendix A

4.1.1Total profit of the system

The objective of the global-system is to maximize the total system profit.

Expected total system profit = Expected profit of remanufacturer *i* + Expected profit of remanufacturer *j* + Expected profit of the retailer

$$MaxE\left[\Pi_g\right] = E\left[\Pi_{Rmi}\right] + E\left[\Pi_{Rmj}\right] + E\left[\Pi_r\right] \tag{1}$$

The profit equation of the retailer can be written as:

Expected profit of the retailer = Expected sales revenue from remanufactured product *i* and *j*

– Cost of purchasing of new remanufactured product *i* and *j*

– Expected cost incurred due to shortage of remanufactured product

+ Expected salvage revenue of left over product *i*

$$\text{Max E}[\Pi_r] = \left(E[\min(Q_i, D_i(p_i, Q_i))](p_i) + E[\min(Q_j, D_j(p_j, Q_j))](p_j)\right) - \left(w_i Q_i + w_j Q_j\right) - \left(SE[D_i(p_i, Q_i) - Q_i]^+ + SE[D_j(p_j, Q_j) - Q_j]^+\right) + \begin{pmatrix} h_i E[Q_i - D_i(p_i, Q_i)]^+ \\ +h_j E[Q_i - D_i(p_i, Q_i)]^+ \end{pmatrix}$$

$$\begin{aligned} E\left[\Pi_r\right] = {} & (\alpha_i - \beta_p p_i + \gamma_p(p_j - p_i) + \beta_s s_i - y_s(s_j - s_i))p_i - w_i(\alpha_i - \beta_p p_i + (p_j - p_i) + \beta_s s_i - \gamma_s(s_j - s_i) + z_i) - S\mu + (p_i + S)(\mu_i - \Theta(z_i)) + h_i(z_i - \mu_i + \Theta(z_i)) \\ & + (\alpha_j - \beta_p p_j + (p_j - p_i)_i + \beta_s s_j - y_s(s_i - s_j))p_j - w_j(\alpha_j - \beta_p p_j + \gamma_p(p_i - p_j) + \beta_s s_j - \gamma_s(s_i - s_j) + z_j) - S\mu + (p_j + S)(\mu_j - \Theta(z_j)) + h_j(z_j - \mu_j + \Theta(z_j)) \end{aligned} \tag{2}$$

Expected total profit of remanufacturer *i* = Sales revenue from remanufactured product *i*

– Expected cost of acquiring the used product

– Cost of remanufacturing product *i*

– Cost incurred to provide the service on new product.

$$\begin{aligned} E\left[\Pi_{Rmi}\right] &= w_i Q_i - Q_i f_i - \sum_{n=1}^{Q_i}\left[a_i + \int_0^1 Q_i \binom{Q_i - 1}{n-1}(c\lambda)\lambda^{n-1}(1-\lambda)^{Q_i - n} d\lambda\right] - \frac{\eta_i s_i^2}{2} \\ &= w_i(\alpha_i - \beta_p p_i + \gamma_p(p_j - p_i) + \beta_s s_i - \gamma_s(s_j - s_i) + z_i) - (\alpha_i - \beta_p p_i + \gamma_p(p_j - p_i) + \beta_s s_i - \gamma_s(s_j - s_i) + z_i)(a_i + \frac{c_i}{2} + f_i) - \frac{\eta_i s_i^2}{2} \end{aligned} \tag{3}$$

Similarly for remanufacturer *j*

$$\begin{aligned} E\left[\Pi_{Rmj}\right] &= w_j Q_j - Q_j f_j - \sum_{n=1}^{Q_j}\left[a_j + \int_0^1 Q_j \binom{Q_j - 1}{n-1}(c\lambda)\lambda^{n-1}(1-\lambda)^{Q_j - n} d\lambda\right] - \frac{\eta_j s_j^2}{2} \\ &= w_j(\alpha_j - \beta_p p_j + \gamma_p(p_i - p_j) + \beta_s s_j - \gamma_s(s_i - s_j) + z_j) - (\alpha_j - \beta_p p_j + \gamma_p(p_i - p_j) + \beta_s s_j - \gamma_s(s_i - s_j) + z_j)(a_j + \frac{c_j}{2} + f_j) - \frac{\eta_j s_j^2}{2} \end{aligned} \tag{4}$$

Since the objective function is concave in nature, it is solved by first-order condition (see Appendix B) and the value of selling price, service level and acquired quantity are as follows.

$$p_i = \frac{(\alpha_i + s_i\beta_s) + (.5c_i + 2f_i)\beta_p + \left(f_i - f_j + .5(c_i - c_j)\right)\gamma_p + (2p_j - a_j)\gamma_p + (s_i - s_j)\gamma_s + (\beta_p + \gamma_p)a_i + (\mu_i - \Theta z_i)}{2(\beta_p + \gamma_p)} \quad (5)$$

$$p_j = \frac{(\alpha_j + s_j\beta_s) + (.5c_j + 2f_j)\beta_p + \left(f_j - f_i + .5(c_j - c_i)\right)\gamma_p + (2p_i - a_i)\gamma_p + (s_j - s_i)\gamma_s + (\beta_p + \gamma_p)a_j + (\mu_j - \Theta z_j)}{2(\beta_p + \gamma_p)} \quad (6)$$

$$F(z_i) = \frac{(p_i + S) - (a_i + \frac{c_i}{2} + f_i)}{(p_i + S - h_i)} \quad (7)$$

$$F(z_j) = \frac{(p_j + S) - (a_j + \frac{c_j}{2} + f_i)}{(p_j + S - h_j)} \quad (8)$$

$$s_i = \frac{\gamma_s(c_j - c_i + 2(p_i - p_j - a_i + a_j)) - \beta_s(c_i + 2(f_i - p_i + a_i))}{2\eta_i} \quad (9)$$

$$s_j = \frac{\gamma_s(c_i - c_j + 2(p_j - p_i - a_j + a_i)) - \beta_s(c_j + 2(f_j - p_j + a_j))}{2\eta_j} \quad (10)$$

The optimal value of p_i and p_j can be obtained by iterative computations and once the optimal value of p_i and p_j are obtained, the corresponding s_i and s_j can also be determined. Finally, the optimal order quantity can be determined as

$$Q_i = z_i * + (\alpha_i - \beta_p p_i * + \gamma_p (p_j - p_i) * + \beta_s s_i * - \gamma_s (s_j * - s_i *))$$

The total channel profit in global situation is derived as follows:

$$\Pi_T = \Pi_g$$

4.2. *Channel wise integrated system*

Here, each remanufacturer cooperates separately with the retailer to form a channel-cooperative system as shown in Fig 1b. Each channel competes with the other channel in terms of setting sale price and corresponding service level of remanufactured product. This model can be considered as a case of inter supply chain competition under intra supply chain cooperation. For example, Canon and Hewlett-Packard are usually sold the toner cartridge through a common retailer in a market and at the same time collect the end-of use cartridges from the market individually. Both the manufacturers try to maximize their individual profit as well as the total channel profit and for that they integrate with the retailer individually.

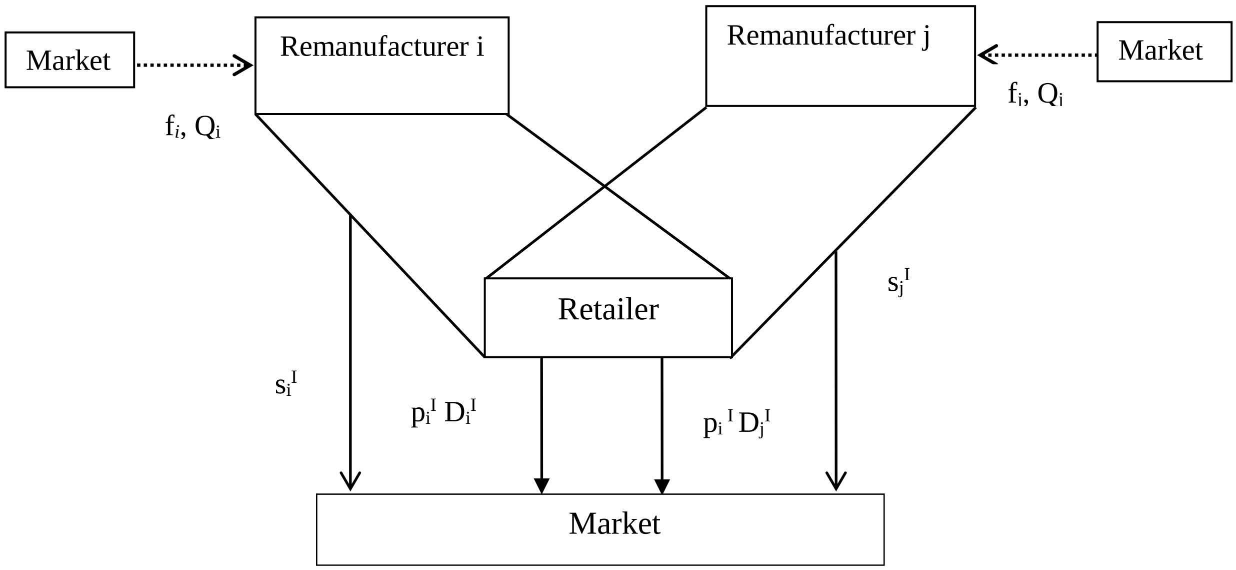

Fig.1b. Channel wise integrated system

The objective function of the first channel is to maximize its total profit

$$MaxE\left[\Pi_{Ii}\right] = E[\Pi_r] + E\left[\Pi_{Rm_i}\right] \tag{11}$$

$$\begin{aligned}\Pi_{Ii} = &(\alpha_i - \beta_p p_i + \gamma_p(p_j - p_i) + \beta_s s_i - y_s(s_j - s_i))p_i - S\mu + (p_i + S)(\mu - \Theta(z_i)) + h_i(z_i - (\mu - \Theta(z_i)) \\ &+(\alpha_j - \beta_p p_j + \gamma_p(p_i - p_j) + \beta_s s_j - y_s(s_i - s_j))p_j - w_j(\alpha_j - \beta_p p_j + \gamma_p(p_i - p_j) + \beta_s s_j - \gamma_s(s_i - s_j) + z_j) - S\mu \\ &+(p_j + S)(\mu - \Theta(z_j)) + h_j(z_j - (\mu - \Theta(z_j)) - (\alpha_i - \beta_p p_i + \gamma_p(p_j - p_i) + \beta_s s_i - \gamma_s(s_j - s_i) + z_i)(a_i + \frac{c_i}{2} + f_i) - \frac{\eta_i s_i^2}{2}\end{aligned} \tag{12}$$

Since the objective function is concave in nature, it is solved by first-order condition (see Appendix C) and the solution is given as

$$p_i = \frac{(\mu - \Theta z_i) + (\alpha_i + s_i\beta_s) + \left(a_i + f_i + .5c_i\right)(\beta_p + \gamma_p) + (2p_j - w_j)\gamma_p + (s_i - s_j)\gamma_s}{2(\beta_p + \gamma_p)} \tag{13}$$

$$F(z_i) = \frac{(p_i + S) - (a_i + \frac{c_i}{2} + f_i)}{(p_i + S - h_i)} \tag{14}$$

$$s_i = \frac{-\beta_s(c_i + 2(f_i - p_i + \theta_i)) - \gamma_s(c_i + 2(f_i - p_i + p_j - w_j + a_i))}{2\eta_i} \tag{15}$$

Solving (13), (14) and (15) simultaneously, one can determine the optimal solution corresponding to the other channel's policy (p_j, z_j, s_j).

The objective function of the second channel is

$$MaxE\left[\Pi_{Ij}\right] = E[\Pi_r] + E\left[\Pi_{Rm_j}\right]$$

Similar to the earlier case, solving the objective function of the second channel, one gets

$$p_j = \frac{(\mu - \Theta z_j) + (\alpha_j + s_j\beta_s) + \left(a_j + f_j + .5c_j\right)(\beta_p + \gamma_p) + (2p_i - w_i)\gamma_p + (s_j - s_i)\gamma_s}{2(\beta_p + \gamma_p)} \tag{16}$$

$$F(z_j)=\frac{(p_j+S)-(a_j+\frac{c_j}{2}+f_j)}{(p_j+S-h_j)} \quad (17)$$

$$s_j=\frac{-\beta_s(c_j+2(f_j-p_j+\theta_j))-\gamma_s(c_j+2(f_j-p_j+p_i-w_i+a_j))}{2\eta_j} \quad (18)$$

Again solving (16),(17) and (18) simultaneously, the optimal solution (p_j,z_j, s_j) can be derived corresponding to the first channel's policy (p_i, z_i, s_i).

The optimal acquired quantity is determined as:

$$Q_i = z_i*+(\alpha_i-\beta_p p_i*+\gamma_p(p_j-p_i)*+\beta_s s_i*-\gamma_s(s_j*-s_i*))$$

The total channel profit in channel wise integrated system situation is derived as follows:

$$\Pi_T=\Pi_{Ii}+\Pi_{Rmj} \; or \; \Pi_T=\Pi_{Ij}+\Pi_{Rmi}$$

4.3. *Decentralized system*

In this case, each member tries to maximize his own profit individually ignoring the impact of his/her decision on other members of the system. On the other hand, to sell the new product, remanufacturer *i* and remanufacturer *j* simultaneously set the wholesale price w_i and w_j. The retailer adds his own margin on the wholesale price to set the final market price p_i and p_j and the system is shown in Fig.1c. Each remanufacturer predicts the wholesale price of his competitor and accordingly sets his own margin on the products. Each remanufacturer provides service directly to the end customer. In this, decentralized system, the remanufacturer can maximize his profit by calculating the corresponding service level given the reaction function of the retailer.

4.3.1.The retailer's problem

The profit equation of the retailer can be written as follows.

Expected retailer's profit = Expected sales revenue from remanufactured product *i* and *j*

– Cost of purchasing of new product

– Expected cost incurred due to shortage of the remanufactured product

+ Expected salvage revenue of left over product *i*

$$\text{Max } E[\Pi_r]=E[\min(Q_i,D_i(p_i,Q_i))](p_i)+E[\min(Q_j,D_j(p_j,Q_j))](p_j)-w_iQ_i-w_jQ_j-SE[D_i(p_i,Q_i)-Q_i]^+ \\ -SE[D_j(p_j,Q_j)-Q_j]^++h_iE[Q_i-D_i(p_i,Q_i)]^++h_jE[Q_j-D_j(p_j,Q_j)]^+$$

$$\Pi_r=(\alpha_i-\beta_p p_i+\gamma_p(p_j-p_i)+\beta_s s_i-y_s(s_j-s_i))p_i+(\alpha_j-\beta_p p_j+\gamma_p(p_i-p_j)+\beta_s s_j-y_s(s_i-s_j))p_j \\ -w_i(\alpha_i-\beta_p p_i+\gamma_p(p_j-p_i)+\beta_s s_i-\gamma_s(s_j-s_i)+z_i)-w_j(\alpha_j-\beta_p p_j+\gamma_p(p_i-p_j)+\beta_s s_j-\gamma_s(s_i-s_j)+z_j) \\ -S\mu+(p_i+S-h_i)(\mu-\Theta(z_i))+h_iz_i-S\mu+(p_j+S-h_j)(\mu-\Theta(z_j))+h_jz_j \quad (19)$$

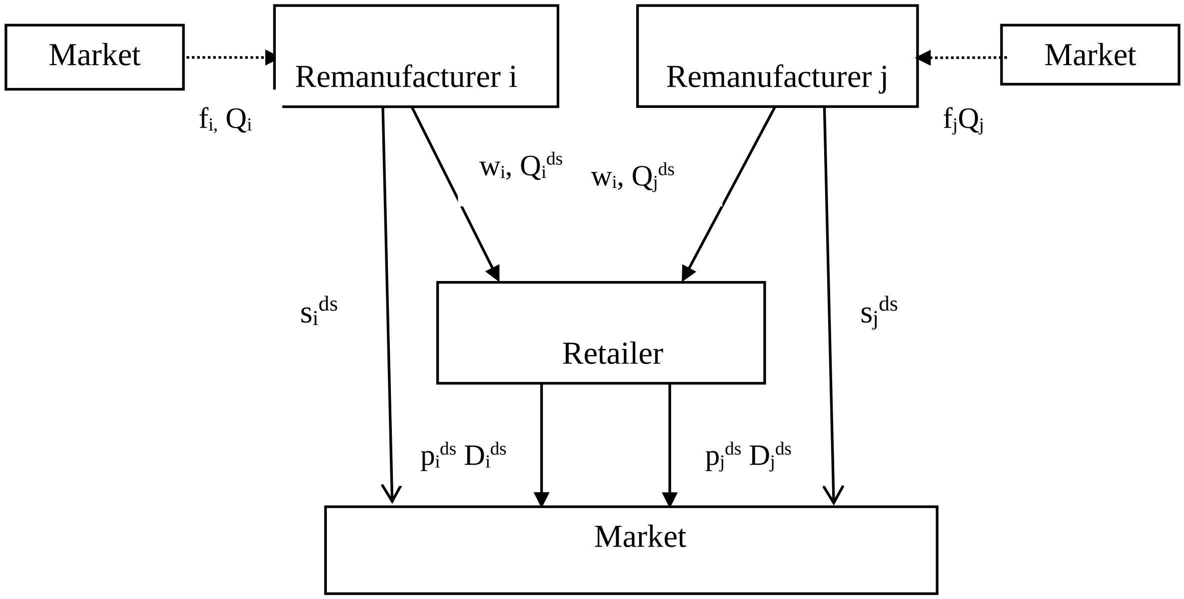

Fig.1c. Decentralized system

Since Π_r is a concave function (proof is given in Appendix D), the simultaneous solution of the first-order condition gives

$$p_i = \frac{\mu - \Theta z_i + \alpha_i + w_i\beta_p + s_i\beta_s + (w_i - w_j + 2p_j)\gamma_p + (s_i - s_j)\gamma_s}{2(\beta_p + \gamma_p)} \quad (20)$$

$$F(z_i) = \frac{(p_i + S) - w_i}{(p_i + S - h_i)} \quad (21)$$

Solving (20), and (21) simultaneously, one can determine the optimal solution corresponding to the other channel's policy (p_j, z_j).

Similar to the earlier case, solving the objective function of the second channel, one gets

$$p_j = \frac{\mu - \Theta z_j + \alpha_j + w_j\beta_p + s_j\beta_s + (w_j - w_i + 2p_i)\gamma_p + (s_j - s_i)\gamma_s}{2(\beta_p + \gamma_p)} \quad (22)$$

$$F(z_j) = \frac{(p_j + S) - w_j}{(p_j + S - h_j)} \quad (23)$$

Solving (22), and (23) simultaneously, the optimal solution can be determined corresponding to the other channel's policy (p_i, z_i).

4.3.2 The remanufacturer's problem

The profit of the remanufacturer can be written as

Expected total profit of remanufacturer *i* = Expected sales revenue from product *i*
– Cost of acquiring the used
– Cost of remanufacturing product *i*
– Service cost

$$\Pi_{Rmi} = w_i Q_i - Q_i f_i - \sum_{n=1}^{Q_i}\left[a_i + \int_0^1 Q_i \binom{Q_i - 1}{n-1} (c\lambda)\lambda^{n-1}(1-\lambda)^{Q_i - n} d\lambda \right] - \frac{\eta_i s_i^2}{2}$$
$$= w_i(\alpha_i - \beta_p p_i + \gamma_p(p_j - p_i) + \beta_s s_i - \gamma_s(s_j - s_i) + z_i) - (\alpha_i - \beta_p p_i + \gamma_p(p_j - p_i) + \beta_s s_i - \gamma_s(s_j - s_i) + z_i)(a_i + \frac{c_i}{2} + f_i) - \frac{\eta_i s_i^2}{2} \quad (24)$$

After solving (20) and (21) simultaneously, we get optimal value of p_i, z_i in terms of s_i. One can put his optimal value in equation (24) and determine the optimal value of s_i through first order differential equation. Finally, one gets the optimal value of service level as follows:

$$s_i^* = \frac{((w_i - (a_i + f_i + 0.5c_i)(\beta_s + \gamma_s)) + \beta_p \gamma_s)}{\eta_i} \quad (25)$$

One can determine the optimal order quantity as

$$Q_i = z_i * + (\alpha_i - \beta_p p_i * + \gamma_p p_j * + \beta_s s_i * - \gamma_s (s_j * - s_i *))$$

The total channel profit in decentralized situation is derived as follows:

$$\Pi_T = \Pi_{Rmi} + \Pi_{Rmj} + \Pi_r$$

4.3. Direct system

In this configuration, remanufactured product is directly sold to the market by the remanufacturer without involving any intermediary and is shown in Fig.1d.

4.3.1. The remanufacturer's problem

The profit of the remanufacturer can be written as

Expected total profit of remanufacturer *i* = Expected sales revenue from product *i*
– Cost of remanufacturing product *i*
– Expected cost incurred due to shortage
– Cost of acquiring the products *i* from the retailer
– Service cost
+ Expected sales revenue from left over product

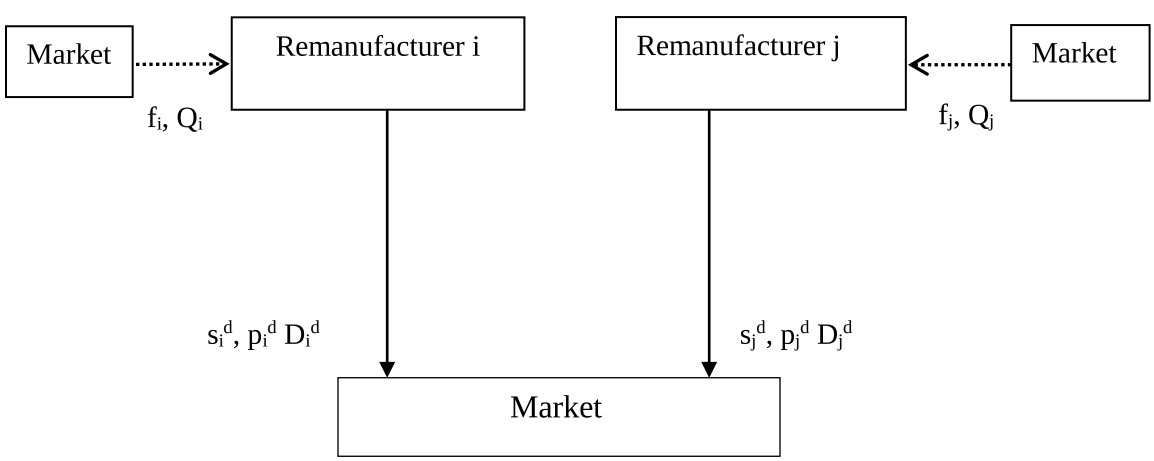

Fig.1d. Direct system

$$
\begin{aligned}
E[\Pi_{Rmi}] &= p_i E[\min(Q_i, D_i(p_i,\varepsilon)] - Q_i f_i - S(D_i - Q_i)^+ - \sum_{n=1}^{Q_i}\left[a_i + \int_0^1 Q_i \binom{Q_i - 1}{n-1}(c\lambda)\lambda^{n-1}(1-\lambda)^{Q_i - n} d\lambda\right] - \frac{\eta_i s_i^2}{2} + h_i(Q_i - D_i)^+ \\
&= p_i(\alpha_i - \beta_p p_i + \gamma_p(p_j - p_i) + \beta_s s_i - \gamma_s(s_j - s_i)) - (\alpha_i - \beta_p p_i + \gamma_p(p_j - p_i) + \beta_s s_i - \gamma_s(s_j - s_i) + z_i) f_i - S\mu + (p_i + S - h_i)E[\min(z_i,\varepsilon)] \\
&\quad - \left(a_i + \frac{c_i}{2}\right)(\alpha_i - \beta_p p_i + \gamma_p p_j + \beta_s s_i - \gamma_s(s_j - s_i) + z_i) + h_i z_i - \frac{\eta_i s_i^2}{2}
\end{aligned} \tag{26}
$$

Since Π_{Rmi} is a concave function (proof is given in Appendix E), the simultaneous solution of the first-order condition gives:

$$p_i = \frac{(\mu - \Theta z_i) + (\alpha_i + f_i\beta_p + s_i\beta_s) + (f_i + p_j)\gamma_p + (\gamma_p + \beta_p).5c_i + (s_i - s_j)\gamma_s + (\beta_p + \gamma_p)a_i}{2(\beta_p + \gamma_p)} \tag{27}$$

$$F(z_i) = \frac{(p_i + S - h_i) - (a_i + \frac{c_i}{2} + f_i)}{(p_i + S - h_i)} \tag{28}$$

$$s_i = \frac{\left(p_i - (f_i + \frac{c_i}{2} + a_i)\right)(\beta_s + \gamma_s)}{\eta_i} \tag{29}$$

Solving (27), (28) and (29) simultaneously, one can determine the optimal solution corresponding to the other channel's policy (p_j, z_j, s_j).

Similar to the previous case, solving the objective function of the second channel, one gets

$$p_j = \frac{(\alpha_j + \gamma_p p_i + \beta_s s_j - \gamma_s(s_i - s_j)) + (\beta_p + \gamma_p)((a_j + \frac{c}{2} + f_j)) + (\mu - \Theta(z_j))}{2(\beta_p + \gamma_p)} \tag{30}$$

$$F(z_j) = \frac{(p_j + S - h_j) - (a_j + \frac{c}{2} + f_j)}{(p_j + S - h_j)} \tag{31}$$

$$s_j = \frac{(\beta_s + \gamma_s)(p_j - (a_j + \frac{c}{2} + f_j))}{\eta_j} \tag{32}$$

Again, solving (30), (31) and (32) simultaneously, one can determine the optimal solution corresponding to the other channel's policy (p_i, z_i, s_i).The optimal acquired order quantity can be determined as follows.

$$Q_i = z_i * + (\alpha_i - \beta_p p_i * + \gamma_p(p_j - p_i)* + \beta_s s_i * - \gamma_s(s_j * - s_i *))$$

5. Computational results

Here, a numerical study is carried out to illustrate the developed models. Demand faced by the retailer is assumed to be price and service sensitive and modeled as follows

$D(p_i,\varepsilon) = \alpha_i - \beta_p p_i + \gamma_p(p_j - p_i) + \beta_s s_i - \gamma_s(s_j - s_i) + \varepsilon,\ (i, j = 1, 2, i \neq j)$. Where, ε is uniformly distributed in the range [0, 100] and the most of the parameters have set to illustrate our model. The various parameters are set as follows:

$$\alpha_i = \alpha_j = 25^2, \beta_p = 5, \gamma_p = 2, \beta_s = 4, \gamma_s = 3, c_i = c_j = 8, \eta_i = \eta_j = 7,$$
$$a_i = a_j = 20, S = 20, w_i = w_j = 80, \text{ and } f_i = f_j = 3 \text{ and } h_i = h_j = h = 15$$

Table2: Results of four different configurations in price and service competition

Parameters	Indirect system			Direct system
	Decentralized	Integrated*	Global	
$p_i = p_j$	95.20	94.90	98.86	88.77
$z_i = z_j$	20.67	87.99	88.45	71.21
$s_i = s_j$	69.43	61.54	41.06	61.77
$Q_i = Q_j$	438	484	384	500
Remanufacturer's profit	6322.00	-	-	15543.00
Retailer's profit	10980.00	-	-	-
Total profit	23612.00	33100.00	36712.00	31085.00

**Channel wise integrated system refers to channel wise coordinated system in all tables and figure*

Table 3: Results of four different configurations for $\gamma_p = 0$ *and* $\gamma_s \neq 0$

Parameters	Indirect system			Direct system
	Decentralized	Integrated*	Global	
$p_i = p_j$	107.6	108.7	88.45	109.1
$z_i = z_j$	72.12	89.45	88.45	76.34
$s_i = s_j$	69.43	69.40	41.06	82.13
$Q_i = Q_j$	385	449	384	485
Retailer's profit	19251.00	-	-	-
Remanufacturer's profit	3934	-	-	13360.00
Total profit	27118.00	32058.00	36712.00	26720.00

Table 4: Results of four different configurations for $\gamma_p \neq 0$ *and* $\gamma_s = 0$

Parameters	Indirect system			Direct system
	Decentralized	Integrated*	Global	
$p_i = p_j$	87.52	83.14	98.86	78.00
$z_i = z_j$	25.47	86.39	88.45	67.46
$s_i = s_j$	46.71	30.08	41.06	46.71
$Q_i = Q_j$	375	424	384	419
Retailer's profit	3663.00	-	-	-
Remanufacturer's profit	12227.00	-	-	16857.00
Total profit	28116.00	36712	35281	33713

Table5: Results of four different configurations when $\gamma_p = 0 \, and \, \gamma_s = 0$

Parameters	Indirect system			Direct system
	Decentralized	**Integrated***	**Global**	
$p_i = p_j$	98.63	96.91	98.86	91.03
$z_i = z_j$	65.21	88.22	88.45	71.88
$s_i = s_j$	46.71	39.95	41.06	36.59
$Q_i = Q_j$	330.00	389	384	388
Retailer's profit	10610.00	-	-	-
Remanufacturer's profit	9849.00	-	-	18120.00
Total profit	30308.00	36802	36710	37241

5.1. *Discussion*

The results in Table 2 show that selling prices of the product are higher in global system under price and service competition compared to direct system. The selling price of both decentralized and channel wise integrated system lies in between the two. The remanufacturer can acquire more products in direct system and less in global system (See Table2). Again service level is high in direct system compared to the global system. The results also show that the retailer receives maximum profit in the decentralized system, whereas remanufacturer's profit is higher in direct system under price and service competition. Further, total profit is considerably high in global system compared to the other three systems. Due to higher selling price in global system, retailer faces lower market demand. As a result, the shortage cost and remanufacturing cost become less compared to other three systems and therefore the overall profit in global system is highest. From the above results; one can infer that the best option will be to maintain global system compared to other three systems.

The results in Table3 show that the selling price of the product are more with higher service level in direct system under service competition whereas, it is less in the global system. The remanufacturer can acquired more used products in direct system (See Table3). Again, the remanufacturer can collect equal number of items in both decentralized and global system and the total profit is highest in global system compared to channel wise integrated and decentralized system. The result also shows that the remanufacturer receives maximum profit in the direct system compared to other systems and, remanufactures will not opt for channel wise integrated system or global system. However, another interesting result state that the total profit in decentralized system is more compared to direct system under service competition because of lower service level cost and lower selling price. More number of the price sensitive customers are attracted because of it and remanufacturer sells more number of remanufactured products in decentralized system compared to direct system. The total profit of global system is higher compared to decentralized system under service competition. From the above analysis, we can infer that best option will be to maintain global systems as compared to other three systems under service competition only.

The results in Table 4 show that the total profit of integrated systems is higher compared to other three systems in the absence of service competition. As observed, in direct system, remanufacturer profit is maximum but at the same time, selling price is less compared to decentralized, global and channel wise integrated systems. The retailer will not take interest to coordinate the channel with each remanufacturer due to loss of profit. The service level is nearly equal in both direct and decentralized system, whereas service level is less in integrated system compared to global system. As a result, remanufacturer expenses low in service level compared to other systems. Therefore, we can infer that best option will be to maintain integrated system as compared to the other three systems in the absence of service competition. The results in table 5 show that the total profit in direct system is higher than global system, decentralized and integrated systems in the absence of price and service competition. The remanufacturer profit is more in direct systems whereas service level and selling price are less compared to global system. For that, the price sensitive customers are attracted more in direct system compared to other systems. As a result, remanufacturer’s sales revenue increases than other. From the above analysis; we can infer that best option will be to adopt direct system as compared to other three systems in the absence of price and service competition.

In view of less profit in the decentralized case, the next pertinent question arises as how to coordinate the channel to improve its profit? In the next section, we have discussed the coordination mechanism to improve the profit of decentralized system.

6. Coordination of the channel

In the competitive business environment, people can buy a greater variety of product at lower price. The retailer faces difficulty to order remanufactured products accurately due to uncertain demand. In Table2, we have observed a scenario in which decentralization system is not preferred by the remanufacturer but is preferred by the retailer due to gain in profit. It is well documented in the literature that coordination overcomes double marginalization problem of decentralized supply chains and also set a final price of the retailer that maximizes the total profit. Many coordination mechanisms are discussed in the literature and here, in order to coordinate the decentralized supply chain, we have considered quantity discount with return contract mechanism. The reason for selection of QD contract is because of its ease of implementation in business transaction. Particularly, QD contract with return are very common in many industries. The retailer opts for the integrated order quantity and selling price when the retailer gets some extra benefits in the form of incentives from the remanufacturer. The retailer's reaction function will satisfy $p_i^d = p_i^g$, and $Q_i^d = Q_i^g$ when new reduced wholesale price w' and buyback price (u) is offered by the remanufacturer to the retailer. The buyback price offered by the remanufacturer here is assumed to be constant irrespective of the unsold quantity returned from the retailer. Remanufacturer is also interested in accepting the unsold products when its profit is not suffered due to the return.

7.Sensitivity analysis and managerial insights

An extensive sensitive analysis is carried out to examine the impact of various parameters on the results.

7.1. Impact of market size (α)

The impact of market size α on total profit is studied and from Fig.3, it is found that the total profit marginally increases as the market size increases in all the four systems considered here. When market size increases, global system makes more profit compared to the other three systems. When market size increases, the demand of remanufactured product also increases, therefore, more revenue will be generated in the global system compared to the other systems due to cooperation among the retailer and the remanufacturers. Again, it is observed that in decentralized system, the total profit is less compared to the direct system up to a certain value of α equal to 800 and is high as the value of market size increases above the certain value 800.Thus, demand level and total revenue is lower compared to decentralized system as market size increases above the threshold value 800. But the total profit in decentralized system is lower than global system. The total profit increases marginally in global system compared to channel wise integrated system. Therefore, the best option will be to maintain global system as compared to other systems.

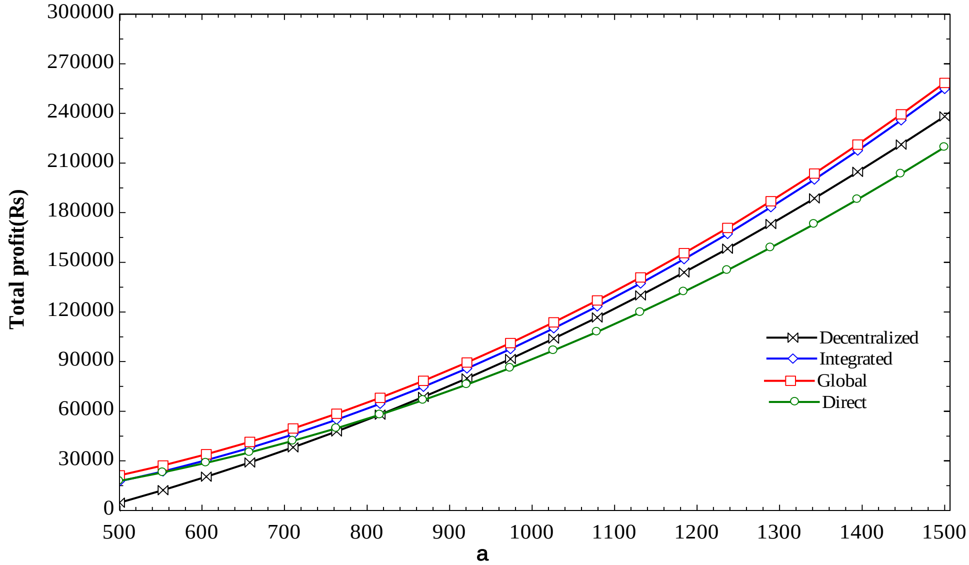

Fig.3.Total profit for the four cases for different values of market size (α)

7.2. Impact of price elasticity(β_p)

Here, the impacts of price elasticity β_p on total profit for all the four systems have been studied. From Fig.4, it is observed that the total profit exponentially decreases as the value of β_p increases in all the four systems. In global system, when β_p increases, it makes more profit compared to other three systems because of channel cooperation between remanufacturers and retailer and inter channel competition. It is again observed that the total profit in channel wise integrated system is more compared to direct system up to a certain value of β_p equal to 7. But the total demand decreases compared to direct system as the value of β_p increases. For that reason, the total profit is less in channel wise integrated system compared to direct system as the value of β_p increases above the certain value 7. Therefore, the best option will be to maintain direct selling system as compared to other systems.

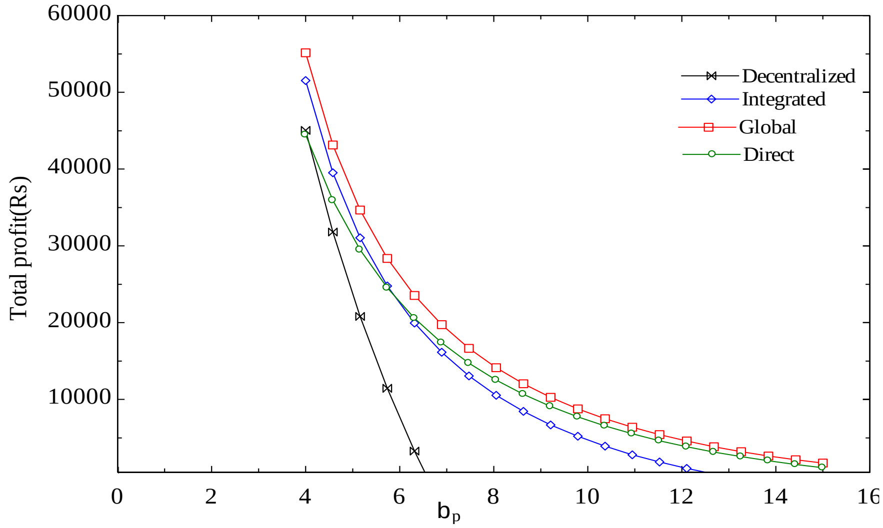

Fig.4.Total profit for the four cases for different values of β_p

7.3. Impact of service elasticity (β_s)

The impact of service elasticity β_s on total profit is examined here. From Fig.5, it is found that the total channel profit increases as the β_s factor increases in channel wise integrated, decentralized and global systems considered here. When β_s increases, global system makes more profit compared to the other three systems. But at the same time, selling price in global system is higher than the other systems. In direct system, the total profit decreases as β_s increases after the value of service elasticity becomes 6.5. It is happened because of high service cost spent by the remanufacturer. Therefore, the best option is to maintain channel wise global system as compared to other systems.

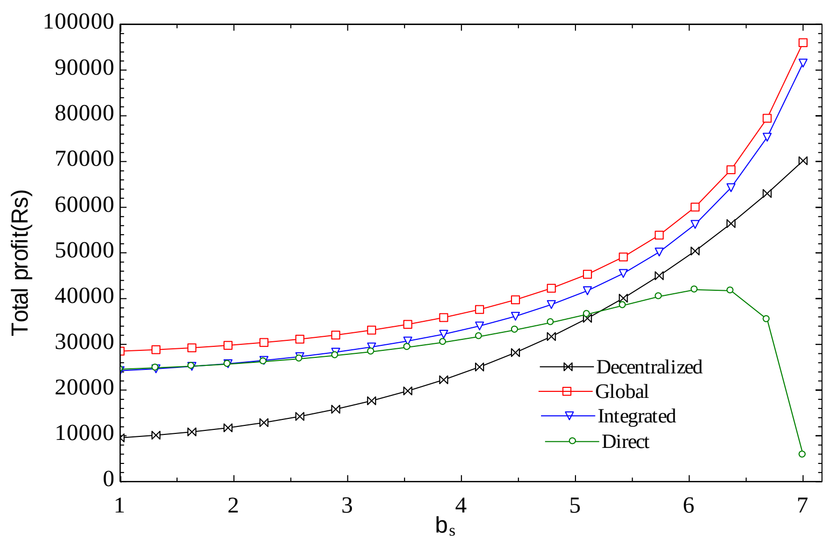

Fig.5.Total profit for the four cases for different values of β_s

7.4. Impact of price competition (γ_p)

The impacts of price competition γ_p on total channel profit for the four cases have been studied here. From Fig.6, it is observed that the total channel profit remains constant, as the price competition increases in global system under price and service competition. Due to cooperation among the remanufacturers and the retailer, the total profit is not affected by the price competition. The total profit decreases in direct, channel wise integrated and decentralized system as price competition increases. In channel wise integrated system, the total profit decreases abruptly compared to direct system after the value of price competition becomes 6.5 because of service competition between the channels. Therefore, the level of service provided by the remanufactures is more importance under price and service competition. In decentralized system, the total profit decreases marginally as price competition increases because of individual price and service competition. Therefore, the best option is to maintain global system as compared to other systems.

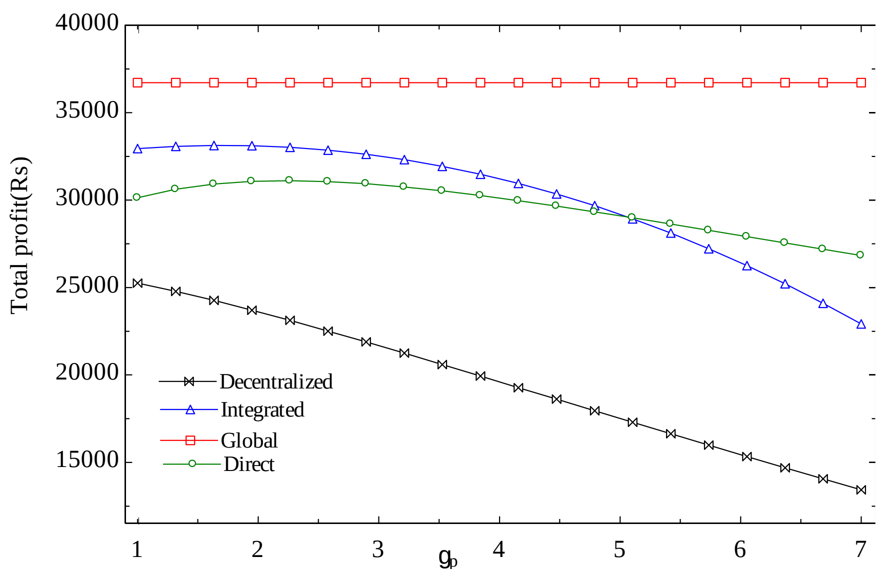

Fig.6.Total profit for the four cases for different values of γ_p

7.5. Impact of service competition (γ_s)

Here, the impacts of service competition γ_s on total channel profit for all the cases have been studied. From Fig.7, it is observed that the total channel profit remains constant, as the service competition increases in global system. Due to cooperation among the remanufacturers and retailers, the total profit is not affected by the service competition in global system. In decentralized case, the total profit decreases marginally as service competition increases. In direct systems, total profit decreases and in channel wise integrated system, total profit also decreases as γ_s increases. In direct system, the remanufacturer directly provides service to the end customer which helps to increase the total profit. Therefore, the level of service is more important under price and service competition. But the rate of service cost increases as the service level increases, and for that the total profit decreases compared to decentralized system after the value of service competition becomes 5.3. In channel wise integrated system, due to the cooperation among the retailer and remanufacturer, the total profit is higher than decentralized and direct systems as the service competition increases. On the other hand, the total system profit is higher in global system as compared to channel wise integrated system.

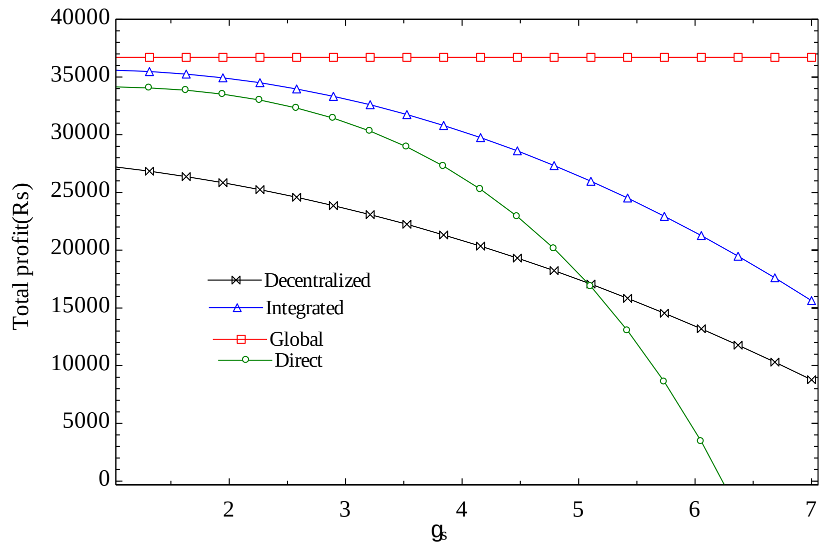

Fig.7. Total profit for the four cases for different values of γ_s

7.6. Impact of variation in demand(ε)

From Fig.8, it is found that for all the four systems considered here, total profit increases as variation in demand increases under certain threshold range. When the range of variation in demand is within the range [0,150], the global system makes more profit compared to the other three systems. But it is interesting to note that when the variation in demand is higher than a certain threshold range [0,150], the direct system makes more profit compared to the other three systems. In both global and channel wise integrated systems, the profit of the remanufacturer and retailer increases exponentially when the range of demand variation is less than 150 and above the range of [0, 150], the value of total profit decreases abruptly due to loss of the retailer and high remanufacturing cost of the remanufacturer. The retailer incurs loss in integrated and global systems due to high demand variation. In direct system, the remanufacturer will produce higher quantity and sell directly at a lower price with a higher service. As a result, the total channel profit increases as variation in demand increases. In case of decentralized system, due to lack of coordination, the retailer orders lesser quantity and sell those products at a lower price. When demand variation increases, the best option will be to maintain direct system compared to other systems.

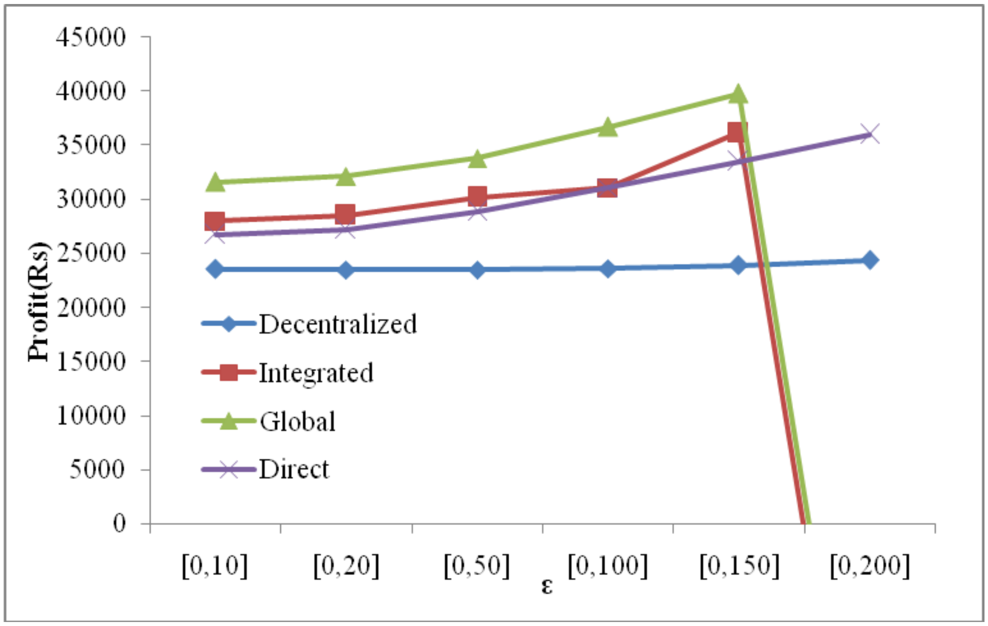

Fig.8.Total profit for the four cases at different values of ε

8. Conclusion and future work

In this paper, we have studied price and service co-opetition in a remanufacturing setup considering uncertain market demands and uncertain conditions of used items. Two different remanufacturers are producing substitutable products and compete on price and service to sell their products directly or through a common retailer. The indirect system is studied for three different remanufacturing configurations (i) Global system (ii) Channel wise integrated system and (iii) Decentralized system. We have shown that total profit will be maximum when the remanufacturers sell their products through a common retailer. This study also presents the condition under which remanufacturers will take interest to sell their products directly. It is observed that the total channel profit in the global system is more compared to direct, integrated and decentralized systems under price and service competition. But in the absence of price and service competition, the channel profit is more under direct system compared to other three systems. The total profit of channel wise integrated system is nearly equal to global system when both price and service competitions are absent. Only in price competition, the channel profit is more in channel-wise integrated system. The total profit in direct system is less compared to channel wise integrated and global systems.

Further, the results reveal that the value of total profit increases as the market size increases for all the systems. It is observed that initially, the total profit marginally decreases as the price elasticity increases. When service elasticity increases, global system makes more profit compared to integrated and decentralized systems due to coordination between the remanufacturers and retailer. The total profit in global system remains constant when price competition increases. In direct systems, when price elasticity increases, it is observed that total profit becomes more than the channel wise integrated system. Therefore, the best option will be to maintain global system compared to other systems in such situations.

There are several directions for future study; a linear demand curve can be modified to include many other possible non-linear curves. Price and service competition among the remanufacturer can be extended to among the retailer under return uncertainty.

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Appendix A

Remanufacturing cost is a linear function of condition ie., $\left(a_i + cX_{(n)}\right)$.

$$\sum_{n=1}^{Q}(a_i + cX_{(n)})$$

$$Q_i\binom{Q_i-1}{n-1}\lambda^n(1-\lambda)^{Q_i-n};\ \lambda\in[0,1]$$

$$\sum_{n=1}^{Q_i}\left[a_i + \int_0^1 Q_i\binom{Q_i-1}{n-1}(c\lambda)\lambda^{n-1}(1-\lambda)^{Q_i-n}d\lambda\right] \qquad \text{(A1)}$$

We know that

$$\int\lambda^k(\alpha+\beta\lambda)^l = \frac{\lambda^{k+1}(\alpha+\beta\lambda)^l}{k+l+1} + \frac{\alpha l}{k+l+1}\int\lambda^k(\alpha+\beta\lambda)^{l-1}$$

$$\text{So}\int_0^1\lambda^n(1-\lambda)^{Q_i-n}d\lambda = \left[\frac{\lambda^{k+1}(1-\lambda)^{Q_i-n}}{Q_i+1}\right]_0^1 + \frac{Q_i-n}{Q_i+1}\int_0^1\lambda^n(1-\lambda)^{Q_i-n-1}d\lambda$$

Which reduces to

$$\int_0^1\lambda^n(1-\lambda)^{Q_i-n}d\lambda = \frac{Q_i-n}{Q_i+1}\int_0^1\lambda^n(1-\lambda)^{Q_i-n-1}d\lambda \qquad (A2)$$

Repeat this application until the exponet of (1-λ)is reduces to zero yields

$$\frac{(Q_i-n)(Q_i-n-1)....(1)}{(Q_i+1)Q_i......(n+2)}\int_0^1\lambda^n d\lambda$$

$$= \frac{(Q_i-n)!(n+1)!}{(Q_i+1)!}\left(\frac{1}{n+1}\right)$$

Then

$$Q_i\binom{Q_i-1}{n-1}\int_0^1\lambda^n(1-\lambda)^{Q_i-n}d\lambda$$

$$= Q_i\binom{Q_i-1}{n-1}\frac{(Q_i-n)!(n+1)!}{(Q_i+1)!}\left(\frac{1}{n+1}\right)$$

$$= \frac{n}{Q_i+1}$$

$$\sum_{n=1}^{D}(a_i + cX_{(n)})$$

$$\sum_{n=1}^{Q_i}\left(a_i + c\frac{n}{Q_i+1}\right) = a_iQ_i + \frac{c(Q_i+1)Q_i}{2(Q_i+1)}$$

$$= a_iQ_i + \frac{cQ_i}{2} \qquad (A3)$$

Proved

Appendix B

$$\text{Max } E[\Pi_r] = E[\min(Q_i, D_i(p_i, Q_i))](p_i - w_i) - SE[D_i(p_i, Q_i) - Q_i]^+ h_i E[Q_i - D_i(p_i, Q_i)]^+$$
$$+E[\min(Q_j, D_j(p_j, Q_j))](p_j - w_j) - SE[D_j(p_j, Q_j) - Q_j]^+ + h_j E[Q_j - D_j(p_j, Q_j)]^+$$

We know that

$$D(p_i, \varepsilon) = \alpha_i - \beta_p p_i + \gamma_p p_j + \beta_s s_i - \gamma_s (s_j - s_i) + \varepsilon \quad \text{(B1)}$$

$$z_i = Q_i - (\alpha_i - \beta_p p_i + \gamma_p (p_j - p_i) + \beta_s s_i - \gamma_s (s_j - s_i)) \quad \text{(B2)}$$

So I can write $Q_i = z_i + (\alpha_i - \beta_p p_i + \gamma_p (p_j - p_i) + \beta_s s_i - \gamma_s (s_j - s_i))$ (B3)

$$E\left[\min(Q_i, D(p_i, \varepsilon))\right] = E\left[\alpha_i - \beta_p p_i + \gamma_p (p_j - p_i) + \beta_s s_i - \gamma_s (s_j - s_i) + \min(z_i, \varepsilon)\right]$$
$$= \alpha_i - \beta_p p_i + \gamma_p (p_j - p_i) + \beta_s s_i - \gamma_s (s_j - s_i) + E[\min(z_i, \varepsilon)]$$

$$E\left[\min(z_i, \varepsilon)\right] = \int_A^z x f(x) dx + \int_{z_i}^B z_i f(x) dx$$
$$= \int_A^B x f(x) dx - \int_{z_i}^B x f(x) dx + \int_z^B z_i f(x) dx$$
$$= \int_A^B x f(x) dx - \int_{z_i}^B (x - z_i) f(x) dx$$

$$\int_{z_i}^B (x - z_i) f(x) dx = \Theta(z_i)$$

$$E\left[D(p_i, \varepsilon) - Q_i\right]^+ = E\left[((\alpha_i - \beta_p p_i + \gamma_p (p_j - p_i) + \beta_s s_i - \gamma_s (s_j - s_i)) - Q_i) + \varepsilon\right]$$
$$= E\left[(\varepsilon - z_i)\right]^+$$
$$= E\left[\varepsilon\right] - E\left[\min(z_i, \varepsilon)\right]$$
$$= \mu - E\left[\min(z_i, \varepsilon)\right]^+$$

So, we can write

$$E\left[\Pi_r\right] = (\alpha_i - \beta_p p_i + \gamma_p (p_j - p_i) + \beta_s s_i - y_s (s_j - s_i)) p_i - w_i (\alpha_i - \beta_p p_i + \gamma_p p_j + \beta_s s_i - \gamma_s (s_j - s_i) + z_i) - S\mu + h_i z_i + (p_i + S - h_i)(\mu - \Theta(z_i))$$
$$+(\alpha_j - \beta_p p_j + \gamma_p (p_i - p_j) + \beta_s s_j - y_s (s_i - s_j)) p_j - w_j (\alpha_j - \beta_p p_j + \gamma_p p_i + \beta_s s_j - \gamma_s (s_i - s_j) + z_j) - S\mu + h_j z_j + (p_j + S - h_j)(\mu - \Theta(z_j)) \quad (B4)$$

$$E\left[\pi_{mi}\right] = w_i Q_i - Q_i f_i - \sum_{n=1}^{Q_i} \left[a_i + \int_0^1 Q_i \binom{Q_i - 1}{n - 1} (c\lambda) \lambda^{n-1} (1 - \lambda)^{Q_i - n} d\lambda \right] - \frac{\eta_i s_i^2}{2} \quad (B5)$$

$$MaxE\left[\Pi_c\right] = E\left[\Pi_{Rmi}\right] + E\left[\Pi_{Rmj}\right] + E\left[\Pi_r\right]$$

$$
\begin{aligned}
\Pi_c = {} & (\alpha_i - \beta_p p_i + \gamma_p (p_j - p_i) + \beta_s s_i - y_s (s_j - s_i)) p_i - w_i (\alpha_i - \beta_p p_i + \gamma_p p_j + \beta_s s_i - \gamma_s (s_j - s_i) + z_i) - S\mu + (p_i + S)(\mu - \Theta(z_i)) \\
& h_i (z_i - (\mu - \Theta(z_i))) + (\alpha_j - \beta_p p_j + \gamma_p (p_i - p_j) + \beta_s s_j - y_s (s_i - s_j)) p_j - w_j (\alpha_j - \beta_p p_j + \gamma_p p_i + \beta_s s_j - \gamma_s (s_i - s_j) + z_j) - S\mu + (p_j + S)(\mu - \Theta(z_j)) \\
& + h_j (z_j - (\mu - \Theta(z_j))) + w_i (\alpha_i - \beta_p p_i + \gamma_p (p_j - p_i) + \beta_s s_i - \gamma_s (s_j - s_i) + z_i) - (\alpha_i - \beta_p p_i + \gamma_p p_j + \beta_s s_i - \gamma_s (s_j - s_i) + z_i)(a_i + \frac{c_i}{2} + f_i) - \frac{\eta_i s_i^2}{2} \\
& + w_j (\alpha_j - \beta_p p_j + \gamma_p (p_i - p_j) + \beta_s s_j - \gamma_s (s_i - s_j) + z_j) - (\alpha_j - \beta_p p_j + \gamma_p p_i + \beta_s s_j - \gamma_s (s_i - s_j) + z_i)(a_j + \frac{c_j}{2} + f_j) - \frac{\eta_j s_j^2}{2} \qquad (B6)
\end{aligned}
$$

$$\frac{\partial \Pi_c}{\partial p_i} = 0$$

$$p_i = \frac{(\alpha_i + s_i \beta_s) + (.5c_i + 2f_i)\beta_p + \left(f_i - f_j + .5(c_i - c_j) \right)\gamma_p + (2p_j - a_j)\gamma_p + (s_i - s_j)\gamma_s + (\beta_p + \gamma_p) a_i + (\mu_i - \Theta z_i)}{2(\beta_p + \gamma_p)} \qquad (B7)$$

Similarly we can findout p_j

$$p_j = \frac{(\alpha_j + s_j \beta_s) + (.5c_j + 2f_j)\beta_p + \left(f_j - f_i + .5(c_j - c_i) \right)\gamma_p + (2p_i - a_i)\gamma_p + (s_j - s_i)\gamma_s + (\beta_p + \gamma_p) a_j + (\mu_j - \Theta z_j)}{2(\beta_p + \gamma_p)} \qquad (B8)$$

$$\frac{\partial \Pi_c}{\partial z_i} = -w_i + (p_i + S - h_i)(1 - F(z_i)) + w_i - (a_i + \frac{c_i}{2} + f_i)$$

$$F(z_i) = \frac{(p_i + S) - (a_i + \frac{c_i}{2} + f_i)}{(p_i + S - h_i)} \qquad (B9)$$

Similarly we can find out

$$F(z_j) = \frac{(p_j + S) - (a_j + \frac{c_j}{2} + f_j)}{(p_j + S - h_j)} \qquad (B10)$$

$$\frac{\partial \Pi_c}{\partial s_i} = 0$$

$$s_i = \frac{\gamma_s (c_j - c_i + 2(p_i - p_j - a_i + a_j)) - \beta_s (c_i + 2(f_i - p_i + a_i))}{2\eta_i} \qquad (B11)$$

Similarly we can find out

$$s_j = \frac{\gamma_s (c_i - c_j + 2(p_j - p_i - a_j + a_i)) - \beta_s (c_j + 2(f_j - p_j + a_j))}{2\eta_j} \qquad (B12)$$

Proved

Appendix C

$$
\begin{aligned}
\Pi_{Ii} = {} & (\alpha_i - \beta_p p_i + \gamma_p (p_j - p_i) + \beta_s s_i - y_s (s_j - s_i)) p_i - w_i (\alpha_i - \beta_p p_i + \gamma_p (p_j - p_i) + \beta_s s_i - \gamma_s (s_j - s_i) + z_i) - S\mu + (p_i + S)(\mu - \Theta(z_i)) \\
& + h_i (z_i - (\mu - \Theta(z_i))) + (\alpha_j - \beta_p p_j + \gamma_p (p_i - p_j) + \beta_s s_j - y_s (s_i - s_j)) p_j - w_j (\alpha_j - \beta_p p_j + \gamma_p (p_j - p_i) + \beta_s s_j - \gamma_s (s_i - s_j) + z_j) - S\mu + (p_j + S)(\mu - \Theta(z_j)) \\
& + h_j (z_j - (\mu - \Theta(z_j))) + w_i (\alpha_i - \beta_p p_i + \gamma_p (p_j - p_i) + \beta_s s_i - \gamma_s (s_j - s_i) + z_i) - (\alpha_i - \beta_p p_i + \gamma_p (p_j - p_i) + \beta_s s_i - \gamma_s (s_j - s_i) + z_i)(a_i + \frac{c_i}{2} + f_i) - \frac{\eta_i s_i^2}{2}
\end{aligned}
$$

$$z_i \in \arg\max_{z_i} \Pi_I (z_{Ii}, p_i, s_i)$$

$$p_i \in \arg\max_{p_i} \Pi_r (p_{Ii}, z_i, s_i)$$

$$s_i \in \arg\max_{s_i} \Pi_r (s_{Ii}, p_i, z_i)$$

$$\frac{\partial^2 \Pi_{Ii}}{\partial {p_i}^2} = -2(\beta_p +)\gamma_p, \frac{\partial \Pi_{Ii}}{\partial p_i \partial z_i} = 1 - \frac{z_i - A}{B - A}, \frac{\partial \Pi_{Ii}}{\partial p_i \partial s_i} = \beta_s + \gamma_s$$

$$\frac{\partial^2 \Pi_{Ii}}{\partial {z_i}^2} = -\frac{S + p_i}{B - A}, \frac{\partial \Pi_{Ii}}{\partial z_i \partial p_i} = 1 - \frac{z_i - A}{B - A}, \frac{\partial \Pi_{Ii}}{\partial z_i \partial s_i} = 0$$

$$\frac{\partial^2 \Pi_{Ii}}{\partial {s_i}^2} = -\eta_s, \frac{\partial \Pi_{Ii}}{\partial s_i \partial p_i} = \beta_s + \gamma_s, \frac{\partial \Pi_{Ii}}{\partial s_i \partial z_i} = 0$$

$$H(p_i, z_i, s_i) = \begin{pmatrix} \frac{\partial^2 \Pi_{Ii}}{\partial {p_i}^2} & \frac{\partial \Pi_{Ii}}{\partial p_i \partial z_i} & \frac{\partial \Pi_{Ii}}{\partial p_i \partial s_i} \\ \frac{\partial \Pi_{Ii}}{\partial z_i \partial p_i} & \frac{\partial^2 \Pi_{Ii}}{\partial {z_i}^2} & \frac{\partial \Pi_{Ii}}{\partial z_i \partial s_i} \\ \frac{\partial \Pi_{Ii}}{\partial s_i \partial p_i} & \frac{\partial \Pi_{Ii}}{\partial s_i \partial z_i} & \frac{\partial^2 \Pi_{Ii}}{\partial {s_1}^2} \end{pmatrix}$$

$$= \begin{pmatrix} -2(\beta_p +)\gamma_p & 1 - \frac{z_i - A}{B - A} & \beta_s + \gamma_s \\ 1 - \frac{z_i - A}{B - A} & -\frac{S + p_i}{B - A} & 0 \\ \beta_s + \gamma_s & 0 & -\eta_s \end{pmatrix}$$

Since, from the operating conditions, determinant is greater than zero and the members of the principal diagonal are negative. Hence, the objective function is a concave function.

$$\frac{\partial \Pi_{Ii}}{\partial p_i} = 0, \frac{\partial \Pi_{Ii}}{\partial z_i} = 0 \, and \, \frac{\partial \Pi_{Ii}}{\partial s_i} = 0$$

$$p_i = \frac{(\mu - \Theta z_i) + (\alpha_i + s_i \beta_s) + (\theta_i + f_i + .5c_i)(\beta_p + \gamma_p) + (2p_j - w_j)\gamma_p + (s_i - s_j)\gamma_s}{2(\beta_p + \gamma_p)} \qquad (G1)$$

$$F(z_i) = \frac{(p_i + S) - (a_i + \frac{c_i}{2} + f_i)}{(p_i + S - h_i)} \qquad (G2)$$

$$s_i = \frac{-\beta_s(c_i + 2(f_i - p_i + \theta_i)) - \gamma_s(c_i + 2(f_i - p_i + p_j - w_j + \theta_i))}{2\eta_i}$$

Appendix D

$$\Pi_r = (\alpha_i - \beta_p p_i + \gamma_p(p_j - p_i) + \beta_s s_i - y_s(s_j - s_i))p_i - w_i(\alpha_i - \beta_p p_i + \gamma_p(p_j - p_i) + \beta_s s_i - \gamma_s(s_j - s_i) + z_i) - S\mu + (p_i + S)(\mu - \Theta(z_i))$$
$$+ (\alpha_j - \beta_p p_j + \gamma_p(p_i - p_j) + \beta_s s_j - y_s(s_i - s_j))p_j - w_j(\alpha_j - \beta_p p_j + \gamma_p(p_i - p_j) + \beta_s s_j - \gamma_s(s_i - s_j) + z_j) - S\mu + (p_j + S)(\mu - \Theta(z_j))$$

$$\frac{\partial \Pi_r}{\partial p_i} = -(\beta_p + \gamma_p)p_i - (\beta_p + \gamma_p)p_i + (\alpha_i + \gamma_p p_j + \beta_s s_i - \gamma_s(s_j - s_i)) + (\mu - \Theta(z_i)) + \gamma_p p_j - w_j \gamma_p + w_i(\beta_p + \gamma_p)$$

$$\frac{\partial^2 \Pi_r}{\partial {p_i}^2} = -2(\beta_p + \gamma_p), \ \frac{\partial \pi_r}{\partial p_i z_i} = 1 - \frac{z_i - A}{B - A}$$

$$\frac{\partial \Pi_r}{\partial z_i} = -w_i + (p_i + s)(1 - F(z_i))$$

$$\frac{\partial^2 \Pi_r}{\partial {z_i}^2} = -\frac{S + p_i}{B - A}, \ \frac{\partial \pi_r}{\partial z_i p_i} = 1 - \frac{z_i - A}{B - A}$$

$$H(p_i, z_i) = \begin{pmatrix} \frac{\partial^2 \Pi_r}{\partial p_i^2} & \frac{\partial \Pi_r}{\partial p_i z_i} \\ \frac{\partial \Pi_r}{\partial z_i p_i} & \frac{\partial^2 \Pi_r}{\partial z_i^2} \end{pmatrix}$$

$$= \begin{pmatrix} -2(\beta_p + \gamma_p) & 1 - \frac{z_i - A}{B - A} \\ 1 - \frac{z_i - A}{B - A} & -\frac{S + p_i}{B - A} \end{pmatrix}$$

$$Where \quad 0 < \frac{z_i - A}{B - A} < 1$$

Since, from the operating conditions, determinant is greater than zero and the members of the principal diagonal are negative. Hence, the objective function is a concave function

$$\frac{\partial \Pi_{Ii}}{\partial p_i} = 0, \frac{\partial \Pi_{Ii}}{\partial z_i} = 0 \, and \, \frac{\partial \Pi_{Ii}}{\partial s_i} = 0$$

$$p_i = \frac{\mu - \Theta z_i + \alpha_i + w_i \beta_p + s_i \beta_s + (w_i - w_j + 2p_j)\gamma_p + (s_i - s_j)\gamma_s}{2(\beta_p + \gamma_p)} \qquad \text{(H1)}$$

$$F(z_i) = \frac{(p_i + S) - w_i}{(p_i + S - h_i)}$$

Appendix E

$$E[\Pi_{Rmi}] = p_i E[\min(Q_i, D_i(p_i, \varepsilon)] - Q_i f_0 - \sum_{n=1}^{Q_i} \left[a_i + \int_0^1 Q_i \binom{Q_i - 1}{n - 1} (c\lambda) \lambda^{n-1} (1 - \lambda)^{Q_i - n} d\lambda \right] - \frac{\eta s_i^2}{2} + h_i (Q_i - D_i)^+$$

$$= p_i(\alpha_i - \beta_p p_i + \gamma_p(p_j - p_i) + \beta_s s_i - \gamma_s(s_j - s_i)) - (\alpha_i - \beta_p p_i + \gamma_p(p_j - p_i) + \beta_s s_i - \gamma_s(s_j - s_i) + z_i) f_i + (p_i - h_i) E[\min(z_i, \varepsilon)]$$
$$- \left(a_i + \frac{c_i}{2} \right) (\alpha_i - \beta_p p_i + \gamma_p(p_j - p_i) + \beta_s s_i - \gamma_s(s_j - s_i) + z_i) + h_i z_i - \frac{\eta_i s_i^2}{2}$$

$$z_i \in \arg\max_{z_i} \Pi_{ch}(z_{Ii}, p_i, s_i)$$
$$p_i \in \arg\max_{p_i} \Pi_r(p_{Ii}, z_i, s_i)$$
$$s_i \in \arg\max_{s_i} \Pi_r(s_{Ii}, p_i, z_i)$$

$$\frac{\partial^2 \Pi_{Rmi}}{\partial p_i^2} = -2(\beta_p + \gamma_p)$$

$$H(p_i, z_i, s_i) = \begin{pmatrix} \frac{\partial^2 \Pi_{Rmi}}{\partial p_i^{\ 2}} & \frac{\partial \Pi_{Rmi}}{\partial p_i \partial z_i} & \frac{\partial \Pi_{Rmi}}{\partial p_i \partial s_i} \\ \frac{\partial \Pi_{Rmi}}{\partial z_i \partial p_i} & \frac{\partial^2 \Pi_{Rmi}}{\partial z_i^{\ 2}} & \frac{\partial \Pi_{Rmi}}{\partial z_i \partial s_i} \\ \frac{\partial \Pi_{Rmi}}{\partial s_i \partial p_i} & \frac{\partial \Pi_{Rmi}}{\partial s_i \partial z_i} & \frac{\partial^2 \Pi_{Rmi}}{\partial s_i^{\ 2}} \end{pmatrix}$$

$$= \begin{pmatrix} -2(\beta_p + \gamma_p) & 1 - \frac{z_i - A}{B - A} & (\beta_s + \gamma_s) \\ 1 - \frac{z_i - A}{B - A} & -\frac{S + p_i}{B - A} & 0 \\ (\beta_s + \gamma_s) & 0 & -\eta_i \end{pmatrix}$$

Since, from the operating conditions, determinant is greater than zero and the members of the principal diagonal are negative. Hence, the objective function is a concave function

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